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1997 J. Phys.: Condens. Matter 9 9597

(http://iopscience.iop.org/0953-8984/9/44/014)

J. Phys.: Condens. Matter **9** (1997) 9597–9600. Printed in the UK PII: S0953-8984(97)83074-3

Scattering of a polaron in the presence of a laser field

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Received 4 April 1997, in final form 15 July 1997

Abstract. An approach is proposed and outlined to study the scattering cross section of a polaron by a weak Coulomb field in the presence of an external single-mode linearly polarized laser field. The frequency of the laser field is assumed to be much larger than the optical phonon frequency. The scattering cross section is found to be more sensitive to the electron–photon interaction than to the electron–phonon interaction.

The problem of the motion of an electron in an ionic crystal has been of interest [1] long since the pioneering work of Fröhlich (1954) [2], where he first developed his model Hamiltonian, which is universally known as the Fröhlich continuum polaron Hamiltonian. The basic assumption in the construction of this Hamiltonian is that the De Broglie wavelength of the electron is very much larger than the interionic separation so that the discrete dielectric lattice may be treated as a continuum medium in which the dressed electron (polaron) behaves like a free particle.

In the present work we propose an approach to calculate the scattering cross sections of the Fröhlich polaron by a weak Coulomb field in the presence of an external single-mode, linearly polarized, homogeneous laser field represented classically by $\varepsilon(t) = \varepsilon \sin(\omega t)$. The corresponding vector potential of the laser field in the Coulomb gauge is given by $\boldsymbol{A}(t) = \boldsymbol{A}_0 \cos(\omega t)$, with $\boldsymbol{A}_0 = c\varepsilon/\omega$, ε and ω being the intensity and frequency of the laser field, and c the velocity of light. This work has relevance to the study of transport phenomena in a dielectric medium in the presence of impurity centres.

The static properties of a polaron in the presence of a Coulomb field have been investigated previously [1]. In the present work we are interested in studying the dynamical properties of such a system in the presence of an external laser field. In fact, although the static properties of a polaron in an external field have been studied extensively [1, 3–5], relatively few theoretical treatments are available for its dynamical properties. Most of the dynamical calculations were confined to the study of transport properties, e.g. electron mobilities and conductivity [6]. To our knowledge, no explicit calculation for the scattering cross sections of a polaron by a Coulomb potential in the presence of an external field has yet been reported.

The frequency of the laser field ω is assumed to be much larger than the phonon (optical) frequency (ω_0) so that the interaction of the laser field with the phonon may be neglected as compared to the electron–laser field interaction. Further, the electrical component of the laser field is kept far below the dielectric breakdown limit. In the present work we have

0953-8984/97/449597+04$19.50

neglected any sort of excitation of the source of the Coulomb field. In other words we consider the elastic channel only.

The modified Fröhlich Hamiltonian for this process is given by [5]

$$H_0 = \left[\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right]^2 + \frac{1}{2}\hbar\omega_0 \sum_{q'} [b^\dagger_{q'} b_{q'} + b_{q'} b^\dagger_{q'}] + 4\pi \mathrm{i}\left(\frac{e^2\hbar}{2\gamma\omega_0 V'}\right)^{1/2} \times \sum_{q'} \frac{1}{q'} [b^\dagger_{q'} \exp(-\mathrm{i}\boldsymbol{q}' \cdot \boldsymbol{r}) - b_{q'} \exp(\mathrm{i}\boldsymbol{q}' \cdot \boldsymbol{r})]. \tag{1}$$

We now scale [2] the energy by $\hbar\omega_0$ and the length by u^{-1} where $u = (2m\omega_0/\hbar)^{1/2}$. The dimensionless Hamiltonian takes the form

$$\tilde{H}_0 = -\tilde{\boldsymbol{\nabla}}^2 + \mathrm{i}\tilde{\boldsymbol{A}} \cdot \tilde{\boldsymbol{\nabla}} + \sum_q b^\dagger_q b_q + \mathrm{i}\left[\frac{4\pi\alpha}{\tilde{V}}\right]^{1/2} \sum_q \frac{1}{q} [b^\dagger_q \exp(-\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{r}) - \mathrm{HC}] \tag{2}$$

where $\tilde{A} = A(2e^2/\hbar\omega_0 mc^2)^{1/2}$ is the dimensionless vector potential, $\tilde{V}$ is the dimensionless volume, $\tilde{\omega}$ and $\tilde{t}$ are respectively the dimensionless frequency and time given by $\omega/\omega_0 = \tilde{\omega}$ and $t\omega_0 = \tilde{t}$, $\tilde{\boldsymbol{\nabla}} = [\hbar/2m\omega_0]^{1/2}\boldsymbol{\nabla}$ and α is the electron–phonon coupling parameter. The full Hamiltonian is thus given by

$$H = H_0 + V_c \tag{3}$$

where V_c (the impurity potential) $= -2\beta/r$, β being the dimensionless Coulomb parameter, treated as a weak perturbation.

The scattering matrix element for such a process (in the first-order theory) is given by [7]

$$T_{if} = -\mathrm{i}\int_{-\infty}^{\infty} \langle\psi_f|V_c|\psi_i\rangle \, \mathrm{d}\tilde{t} \tag{4}$$

where ψ_i and ψ_f are the initial and final states of the dressed polaron which are given by solutions of $\tilde{H}_0\psi = \tilde{E}\psi$ for weak electron–phonon interaction [8] (LLP), given by

$$\psi = \psi_0\left(1 - \mathrm{i}\left(\frac{4\pi\alpha}{V}\right)^{1/2} \sum_q \frac{\exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r})b^\dagger_q}{q(\boldsymbol{p}-\boldsymbol{q})^2 + 1 - p^2}\right)\Pi_q|0_q\rangle. \tag{5}$$

In fact the above wavefunction is only the perturbative limit of the variational wavefunction of Lee, Low and Pines (LLP) [8]. In (5) ψ_0 is the solution of the following equation [9]:

$$[-\tilde{\boldsymbol{\nabla}}^2 + \mathrm{i}(\tilde{\boldsymbol{A}} \cdot \tilde{\boldsymbol{\nabla}})]\psi_0 = \mathrm{i}\frac{\partial\psi_0}{\partial\tilde{t}}$$

and is given by [7, 9]

$$\psi_0 \sim \exp[\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{r} + \mathrm{i}\boldsymbol{p}\cdot\alpha_0 \sin\tilde{\omega}\tilde{t} - \tilde{E}\tilde{t})] \tag{6}$$

where $\alpha_0 = (e\varepsilon/m\omega^2)[2m\omega_0/\hbar]^{1/2}$ and p is the dimensionless polaron momentum. The solution ψ_0 is the Volkov solution [7, 9] representing the non-relativistic wavefunction for the free electron in a laser field.

The first-order dimensionless polaron energy is chosen as [7]

$$\begin{aligned} \tilde{E} &= p^2 - \frac{4\pi\alpha}{\tilde{V}} \sum \frac{1}{q^2[(\boldsymbol{p}-\boldsymbol{q})^2 + 1 - p^2]} = p^2 - \alpha\frac{\sin^{-1} p}{p} \\ &= -\alpha + p^2(1 - \alpha/6) + O(p^4) \qquad \text{for } p < 1, \alpha \to 0 \\ &= p^2 - \frac{\pi\alpha}{2p} \qquad \text{for } p \geqslant 1, \alpha \to 0. \end{aligned} \tag{7}$$

In view of the equations (3)–(5), the matrix element (4) reduces to the form

$$T_{if} = \tilde{a} \int \sum_{\ell=-\infty}^{\infty} \left[-\frac{2\beta}{r} \exp(-\mathrm{i}\boldsymbol{Q}\cdot\boldsymbol{r}) J_\ell(\boldsymbol{Q}\cdot\boldsymbol{\alpha}_0)\delta(E_f - E_i + \ell\tilde{\omega}) \right] \times \left[1 + \frac{4\pi\alpha}{\tilde{V}} \sum_q \frac{1}{\boldsymbol{q}^2[(\boldsymbol{p}_f - \boldsymbol{q})^2 + 1 - p_f^2][(\boldsymbol{p}_i - \boldsymbol{q})^2 + 1 - p_i^2]} \right] \mathrm{d}\boldsymbol{r} \tag{8}$$

where $\boldsymbol{Q} = \boldsymbol{p}_f - \boldsymbol{p}_i$ and $\tilde{a}$ is some dimensionless constant; $J_\ell(x)$ denotes the Bessel function with argument x. We now convert the summation over $\boldsymbol{q}$ to integration and then perform the integration over $\boldsymbol{r}$ to obtain (apart from some numerical constants)

$$T_{if} \sim (1/Q^2) \sum_{\ell=-\infty}^{\infty} J_\ell(\boldsymbol{Q}\cdot\boldsymbol{\alpha}_0)\delta(E_f - E_i + \ell\tilde{\omega})[1 + (2\pi)^{-3} 4\pi\alpha L] \tag{9}$$

where

$$L = 1/(B^2 - AC)^{1/2} \ln[B + (B^2 - AC)^{1/2}/[B - (B^2 - AC)^{1/2}]]$$

with

$$AC = 2[1 - \boldsymbol{p}_i\cdot\boldsymbol{p}_f + \mathrm{i}\{(1 - p_i^2)(p_f^2 - 1)\}^{1/2}]$$
$$B = (1 - p_i^2)^{1/2} + \mathrm{i}(p_f^2 - 1)^{1/2}.$$

The differential cross section for this process is given by

$$\sigma(\theta) = \frac{p_f}{p_i} |T_{if}|^2. \tag{10}$$

The total cross section is obtained by integrating the differential cross section over the solid angle;

$$\sigma_t = 2\pi \int_0^\pi \sigma(\theta) \sin\theta \, \mathrm{d}\theta. \tag{11}$$

The conservation of energy is given by

$$\tilde{E}_f = \tilde{E}_i \pm \ell\tilde{\omega} \tag{12}$$

where $\tilde{E}_i$ and $\tilde{E}_f$ are the initial- and final-state energies of the dressed polaron, which can be obtained from (7) and (12) for some fixed p_i, and ℓ is the quantum number of the photon absorbed from the laser field ($\ell = +1$ corresponds to one-photon absorption and $\ell = -1$ corresponds to one-photon emission). In this note we report only the results due to absorption of one photon i.e., $\ell = +1$, $\tilde{\omega} \gg 1$ (i.e., $\omega \gg \omega_0$) and for low initial polaron momentum ($p_i < 1$). The final polaron momentum is obtained from (7) and (12).

Table 1 displays the total scattering cross sections (TCSs) for different values of α, α_0, β and p_i with $\tilde{\omega} = 100$. It may be noted from the table that the TCS increases with increasing Coulomb binding parameter (β) for fixed values of α, α_0 and p_i. Moreover the TCS is found to be much more sensitive to the electron–photon coupling (α_0) than to the electron–phonon coupling (α) parameter. It is also evident from table 1 that an increase of the strength of the external laser field suppresses the TCS. For fixed values of α, α_0 and β the TCS decreases with increasing polaron momentum p_i as expected. The detailed results (both differential and total cross sections) and their analysis will be communicated in a future work.

Finally we would like to comment that, as a first attempt, the present model treats the polaron perturbatively for simplicity. However for realistic situations, one should consider the exact variational wavefunction for the polaron.

Table 1. Laser assisted elastic cross sections (total) for electron in a dielectric medium in units of $u^{-2}[u = \sqrt{(2m\omega/\hbar)}]$ for $\tilde{\omega} = 100$ ($\tilde{\omega} = \omega/\omega_0$), $\alpha_0 = 0.365$, and $\alpha_0 = 3.65$ (data within []). The numbers in parentheses indicate the power of 10 by which the entry is to be multiplied.

	$\alpha = 0.1$		$\alpha = 0.5$	
p_i	$\beta = 0.1$	$\beta = 0.5$	$\beta = 0.1$	$\beta = 0.5$
0.1	8.94(−5)	2.23(−3)	9.12(−5)	2.28(−3)
	[2.46(−5)]	[6.15(−4)]	[2.50(−5)]	[6.25(−4)]
0.5	1.80(−5)	4.50(−4)	1.84(−5)	4.59(−4)
	[2.84(−6)]	[7.11(−5)]	[2.92(−6)]	[7.29(−5)]
0.9	1.02(−5)	2.55(−4)	1.08(−5)	2.69(−4)
	[1.86(−6)]	[4.65(−5)]	[1.91(−6)]	[4.76(−5)]

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